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Interpolating the ’t Hooft Model Between IFD and LFD in the Coulomb Gauge

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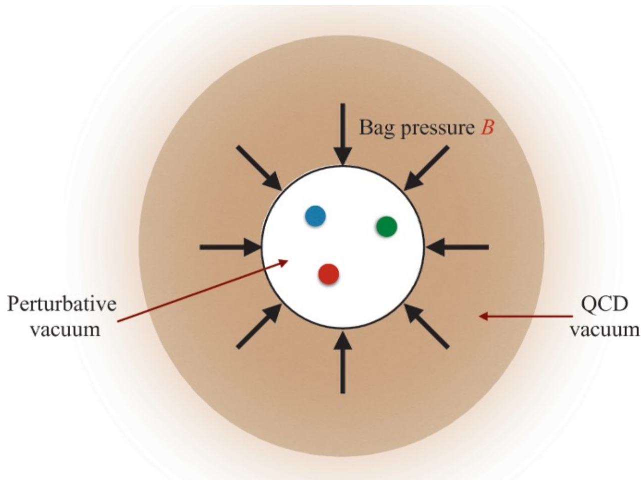
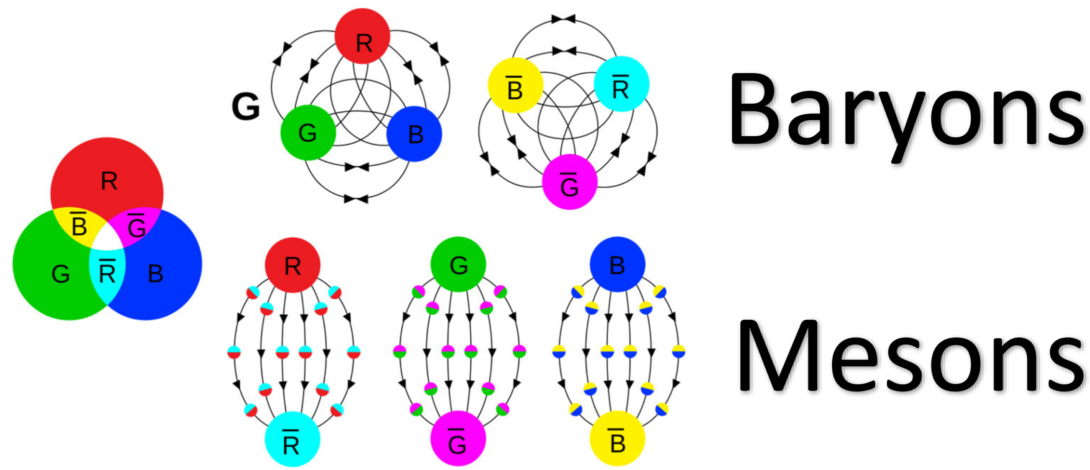
Outline:

- Short QCD introduction
- Introduction to Light Front Dynamics
- Interpolation between instant and front forms
- Motivation behind Interpolation
- ‘t Hooft’s toy meson model
- Why use the Coulomb gauge?
- Mass gap
- Mass Spectra
- An interesting application

Quark Model in QCD



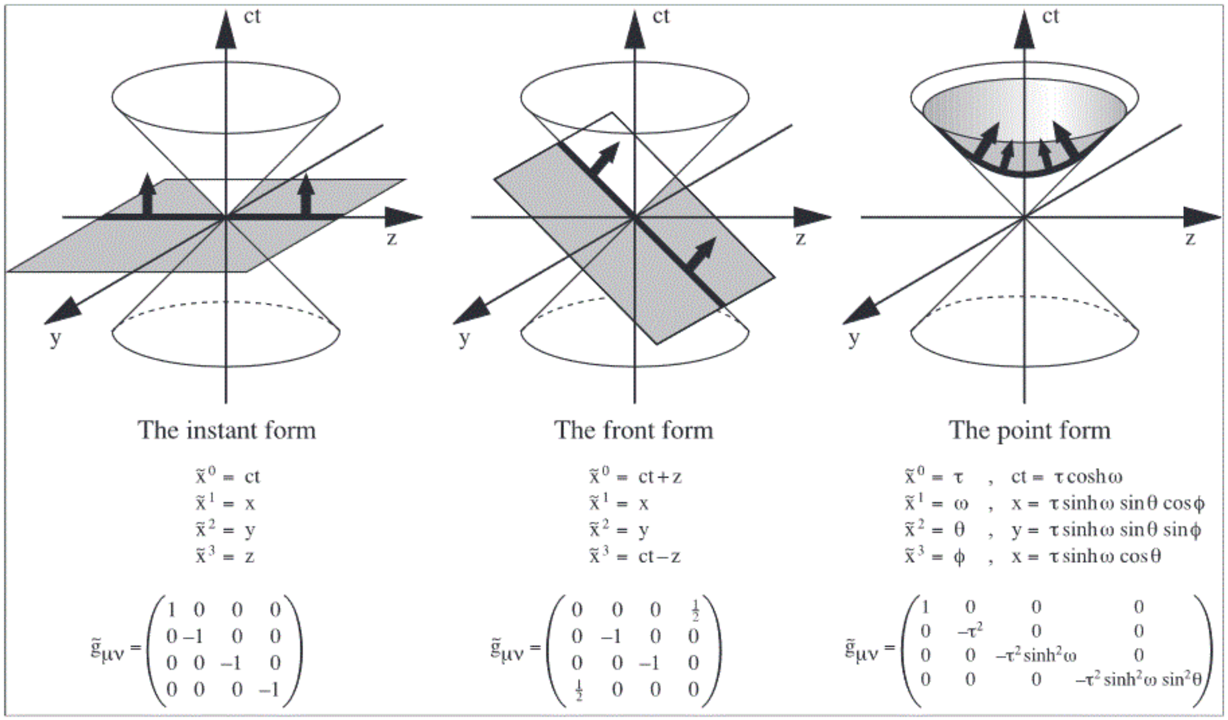
[1] MIT Bag Model



[2] QCD Illustration

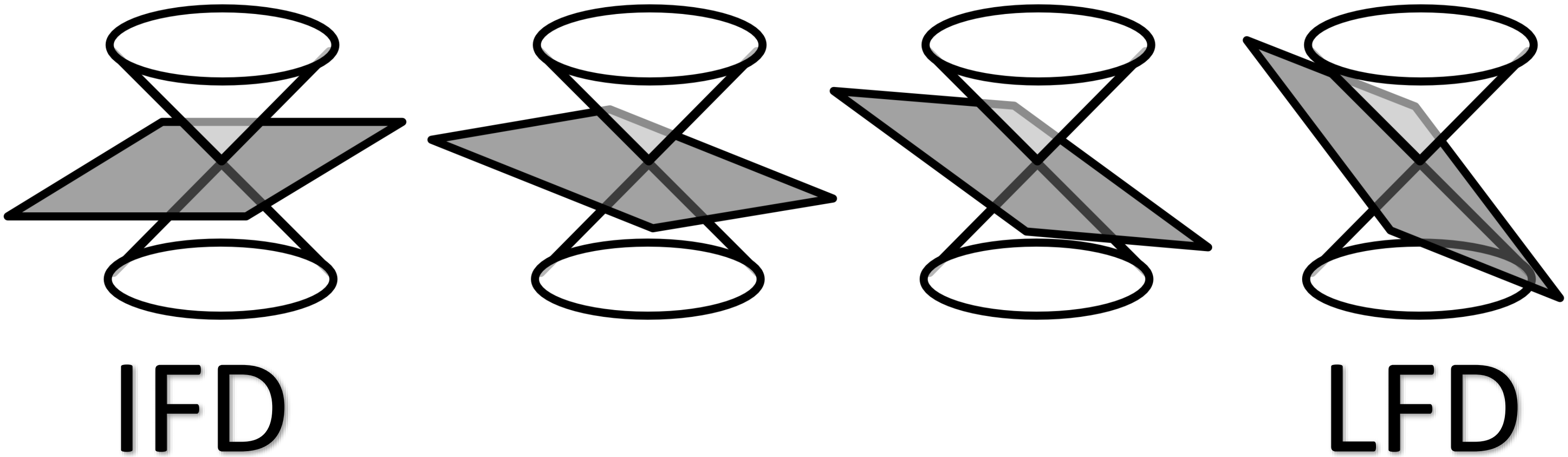
- Baryons are an Nc valence quark bound state
- Mesons are valence quark-antiquark bound states
- Running coupling constant unlike QED

Light Front Dynamics

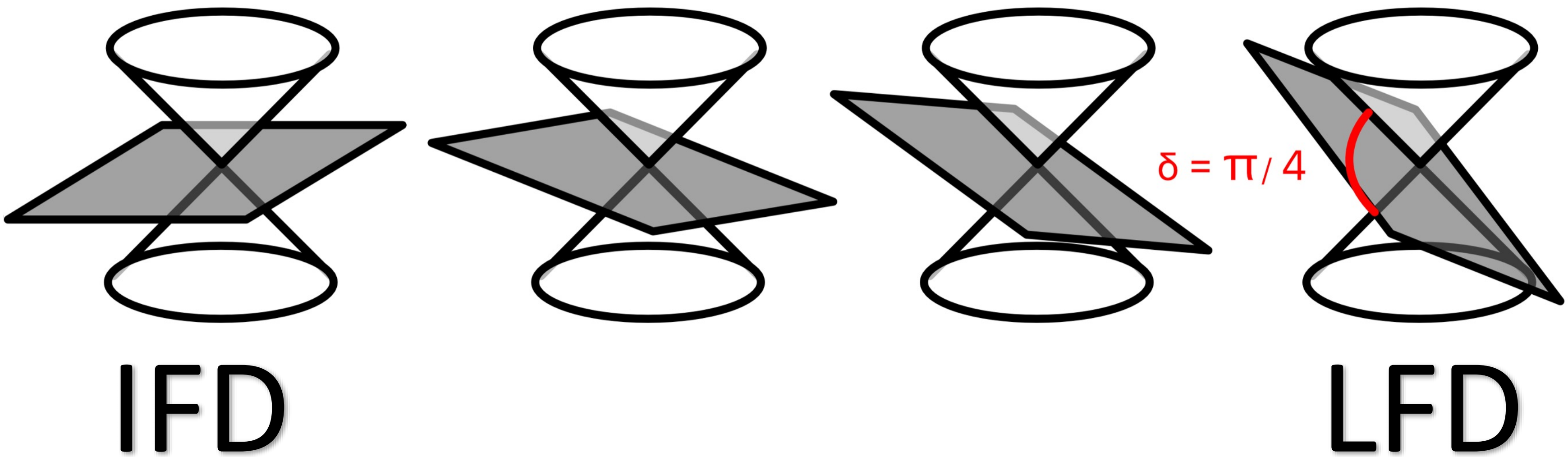


[3] Illustration of Dirac's proposed coordinates

Interpolation between Instant and Front Form



Interpolation between Instant and Front Form



‘t Hooft’s Meson Model

$$\mathcal{L} = -\frac{1}{4} F^{\hat{\mu}\hat{\nu}} F_{\hat{\mu}\hat{\nu}} + \bar{\psi}\left(i\gamma^{\hat{\mu}} D_{\hat{\mu}} - m\right)\psi$$

$$D_{\hat{\mu}} = \partial_{\hat{\mu}} - igA^a_{\hat{\mu}} t_a$$

$$F^a_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} A^a_{\hat{\nu}} - \partial_{\hat{\nu}} A^a_{\hat{\mu}} + g f^{abc} A^b_{\hat{\mu}} A^c_{\hat{\nu}}$$

- 1 space dimension, 1 time dimension
- Large Nc limit
- Predicts color confinement
- Coupling constant has mass dimension

Motivation

$$g^{\hat{\mu}\hat{\nu}} = g_{\hat{\mu}\hat{\nu}} = \begin{bmatrix} cos(2\delta) & sin(2\delta) \\ sin(2\delta) & -cos(2\delta) \end{bmatrix}$$

- To establish a formal link between IFD and LFD
- Simplify calculations ordinarily too difficult to solve in IFD
- Explore an alternative quasi-PDF implementable in Lattice QCD

Why Coulomb Gauge?

*In IFD 1+1

$$\partial_\mu A^\mu = 0 \Longrightarrow \begin{cases} A^0 = 0 & \text{:= Coulomb Gauge} \\ \text{OR} & \\ A^1 = 0 & \text{:= Axial Gauge} \end{cases}$$

Lorentz Condition

- Axial gauge calculations have already been done [2]
- Consistent with QED
- All degrees of freedom are physical
- No conjugate gauge field

Conjugate Field

$$\Pi^0(x) = \frac{\partial \mathcal{L}}{\partial \left(\partial_{\hat{+}} A^0\right)} = 0 \Longrightarrow \left[A^0, \Pi^0\right] = 0$$

- Conjugate gauge field commutes with the gauge field!
- Simplifies calculations

Comparison of Axial and Coulomb Gauge

Lagrangian

Axial Gauge:

$$\mathcal{L} = \frac{1}{2}\left(\partial_{\hat{-}} A^{a}_{\hat{+}}\right)^{2} + \bar{\psi}\left[i\gamma^{\hat{+}} D_{\hat{+}} + i\gamma^{\hat{-}}\partial_{\hat{-}} - m\right]\psi$$

Coulomb Gauge:

$$\mathcal{L} = \frac{1}{2}\left(\partial_{\hat{+}} A^{a}_{\hat{-}}\right)^{2} + \bar{\psi}\left[i\gamma^{\hat{-}} D_{\hat{-}} + i\gamma^{\hat{+}}\partial_{\hat{+}} - m\right]\psi$$

Comparison of Axial and Coulomb Gauge

Axial Gauge:

- The field equation is dependent on the interpolating space dimension.
- The source of the potential is explained by an effective charge density, which acts like a background field.

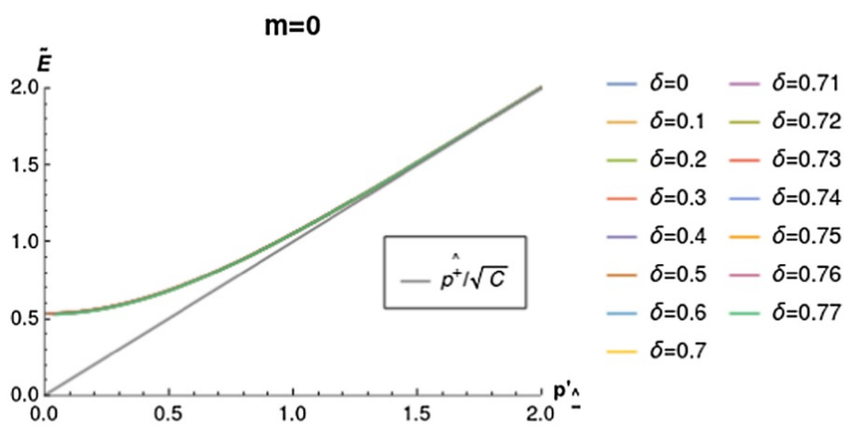
Coulomb Gauge:

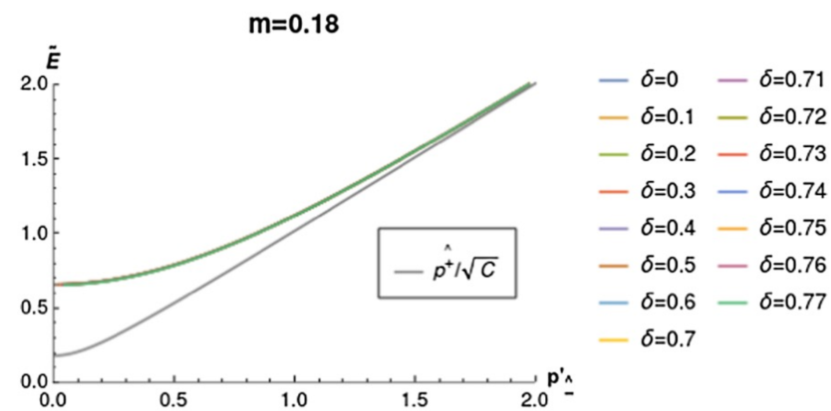
- The field equation is dependent on the interpolating time dimension.
- The source of the potential is explained by an effective current density, which acts like a globally changing field through time.

$$\partial^\mu A_\mu = J^\mu$$

Expected Mass Gap Solution

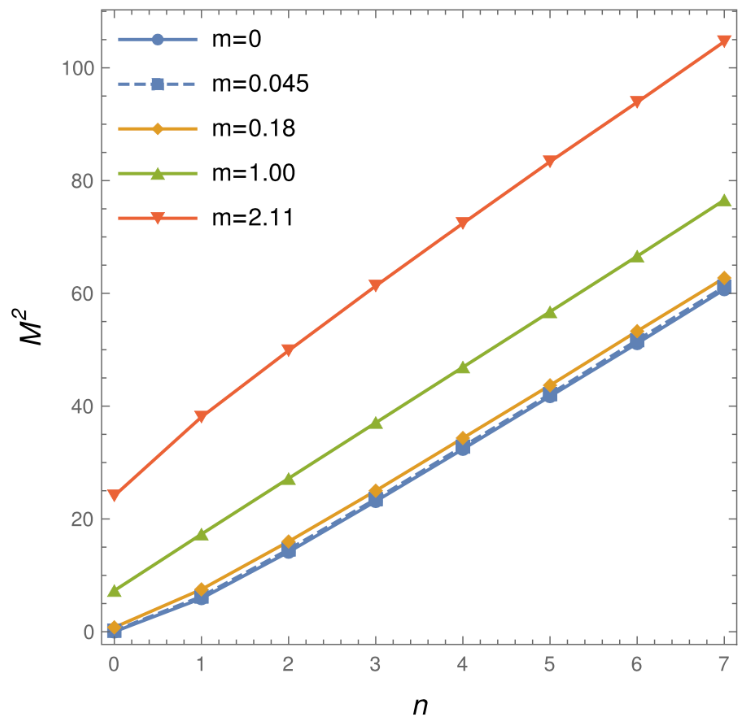
$$\tilde{E}\left(p'_{\hat{-}}\right)^2 = p'^2_{\hat{-}} + M\left(p'_{\hat{-}}\right)^2$$





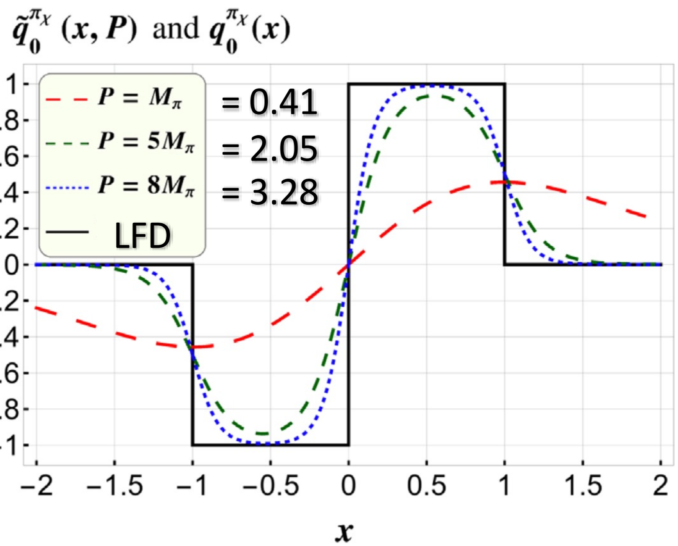
Note: This is independent of the interpolation angle

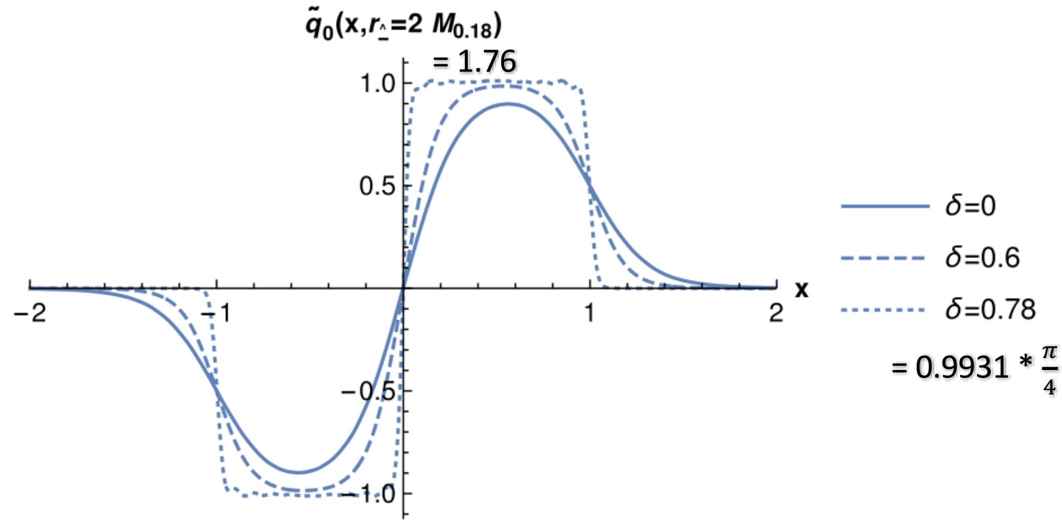
Mass Spectra and Regge Behavior



- Mass increases (roughly) linearly with principal quantum number

Application: Alternative Quasi-PDFs





Jia, Y., Liang, S., Xiong, X., & Yu, R. (2018). ARXIV.1804.04644

- Dependent on Interpolation angle and momentum
- Does not suffer from the large momentum boost

Conclusion and Future Work

Conclusion:

- The ‘t Hooft model is interpolated between the instant form and the front form in the axial gauge, looking to explore the Coulomb gauge
- The mass gap equation is solved and plotted
- The mass spectra shows Regge behavior, which we see in nature
- Bound state wave functions are used to calculate alternative quasi-PDFs

Future work:

- (3+1) and Nc = 3 needs to be explored
- Testing the alternative quasi-PDF on the lattice
- Exploring the time-like region using LFD

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Images:
[1] Hoyer, P. (2021). Journey to the Bound States. arXiv. https://doi.org/10.48550/ARXIV.2101.06721
[2] https://webific.ific.uv.es/web/en/content/lattice-qcd-numerical-approach-strong-force
[3] https://ncatlab.org/nlab/show/light-cone+quantization
[Else] Ma, B., & Ji, C.-R. (2021). Interpolating ’t Hooft model between instant and front forms. In Physical Review D (Vol. 104, Issue 3). American Physical Society (APS). https://doi.org/10.1103/physrevd.104.036004